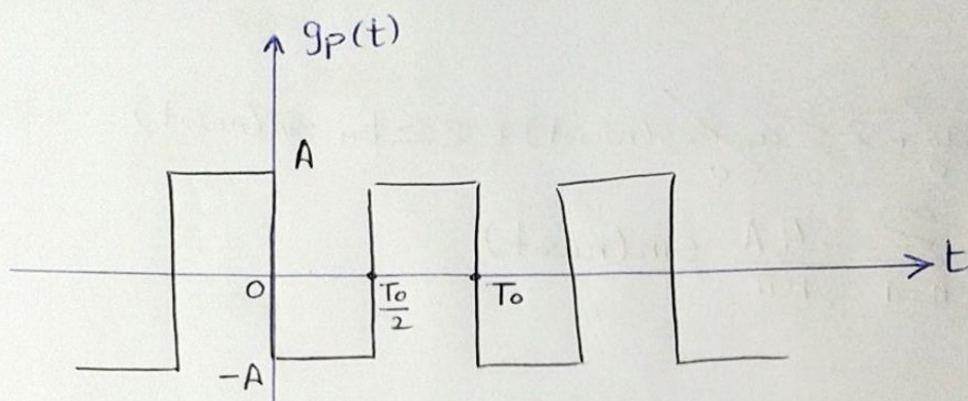


Quiz # 1

Sol.

①



• T_0 from 0 to T_0

$$g_p(t) = \begin{cases} -A & 0 < t < T_0/2 \\ +A & T_0/2 < t < T_0 \end{cases}$$

• $g_p(t)$ is odd, $a_0 = 0$ $a_n = 0$

$$b_n = \frac{1}{T_0} \int_0^{T_0} g_p(t) \sin(n\omega_0 t) dt$$

$$= \frac{1}{T_0} \left[\int_0^{T_0/2} -A \sin(n\omega_0 t) dt + \int_{T_0/2}^{T_0} A \sin(n\omega_0 t) dt \right]$$

$$= \frac{A}{n\omega_0 T_0} \left[\cos(n\omega_0 t) \Big|_0^{T_0/2} + \cos(n\omega_0 t) \Big|_{T_0/2}^{T_0} \right]$$

$$= \frac{A}{2n\pi} \left[\cos(n\pi) - \cos(0) + \cos(n\omega_0 \frac{T_0}{2}) - \cos(n\omega_0 T_0) \right]$$

$$= \frac{A}{2n\pi} [2 \cos(n\pi) - 2]$$

$$= \frac{A}{n\pi} [\cos(n\pi) - 1] \quad , \quad \cos(n\pi) = \begin{cases} 1, & n \text{ even} \\ -1, & n \text{ odd} \end{cases}$$

$$\therefore b_n = \begin{cases} -\frac{2A}{n\pi}, & n \text{ odd} \\ 0, & n \text{ even} \end{cases}$$

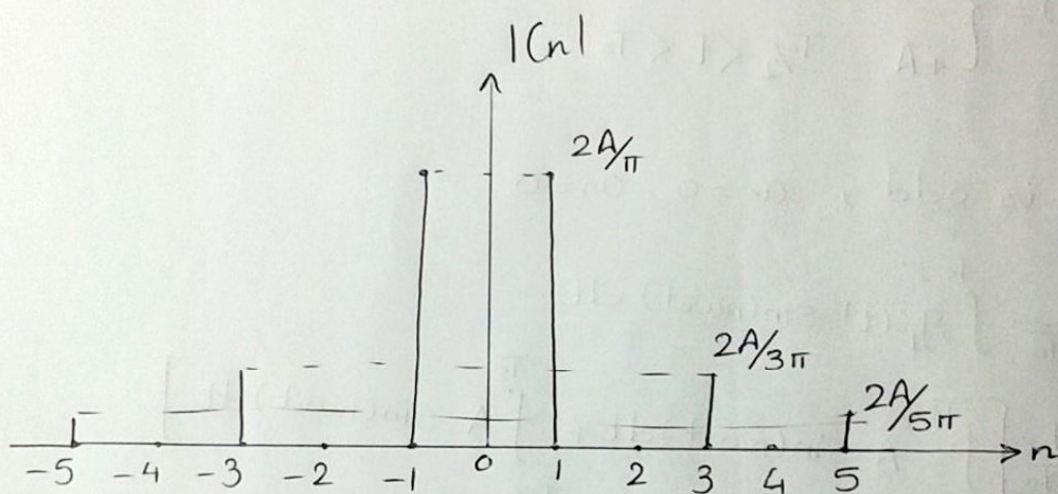
①

$$|C_n| = \sqrt{a_n^2 + b_n^2} = |b_n|$$

∴ F.S.

$$\begin{aligned} g_p(t) &= a_0 + 2 \sum_{n=1}^{\infty} a_n \cos(n\omega_0 t) + 2 \sum_{n=1}^{\infty} b_n \sin(n\omega_0 t) \\ &= \sum_{n=1}^{\infty} \frac{-4A}{n\pi} \sin(n\omega_0 t) \end{aligned}$$

$$|C_n| = \frac{2A}{n\pi}, \quad n \text{ odd}$$



2) i) $g(t) = 100$

$$G(f) = 100 \delta(f)$$

using duality $\begin{matrix} \delta(t) \Rightarrow 1 \\ 1 \Rightarrow \delta(f) \end{matrix}$

ii) $g(t) = \text{rect}\left(\frac{t-2}{4}\right) + 8 \sin(6\pi t)$

$A \text{rect}\left(\frac{t-t_0}{T}\right) \Rightarrow AT \text{sinc}(fT) \cdot e^{-j2\pi f t_0} \rightarrow$ using time shift
 $\sin(2\pi f_c t) \Rightarrow \frac{A}{2j} [\delta(f-f_c) - \delta(f+f_c)] \rightarrow$ using freq. shift

$$\therefore G(f) = 4 \text{sinc}(4f) \cdot e^{-j4\pi f} + \frac{8}{2j} [\delta(f-3) - \delta(f+3)]$$

(2)

(5)

$$\text{iii) } g(t) = 3 \operatorname{sgn}(t-3)$$

$$\operatorname{sgn}(t) \Longleftrightarrow \frac{1}{j\pi f}$$

$$G(f) = \frac{3}{j\pi f} \cdot e^{-j2\pi f(3)} \longrightarrow \text{using time shift}$$